HW: p467 #2-24 evens

p467

~~21~~, 23, ~~17~~, ~~18~~, 11, 13,

11 $\lim_{b\to-\infty}\int_b^{-2}\frac{2\,dx}{x^2-1} = \lim_{b\to-\infty}\int_b^{-2}\frac{2}{(x+1)(x-1)}\,dx$

$\frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = \lim_{b\to-\infty}\int_b^{-2}\frac{-1}{x+1} + \frac{1}{x-1}\,dx$

$2 = A(x-1) + B(x+1)$

$A = -1 \qquad B = 1$

$= \lim_{b\to-\infty}\Big[-\ln|x+1| + \ln|x-1|\Big]_b^{-2}$

$= \lim_{b\to-\infty}\Big[\ln\Big|\frac{x-1}{x+1}\Big|\Big]_b^{-2}$

$= \lim_{b\to-\infty}\left(\ln\frac{-3}{-1}\right) - \ln\left|\frac{b-1}{b+1}\right|$ (1)

$\ln 3 - \ln 1$

$\ln 3 - 0 = \ln 3$

13 $\lim_{b\to\infty} \int_{-1}^{b} \frac{dx}{x^2+5x+6} = \lim_{b\to\infty} \int_{-1}^{b} \frac{-1}{x+3} + \frac{1}{x+2}\, dx$

$\frac{1}{x^2+5x+6} = \frac{A}{(x+3)} + \frac{B}{x+2}$

$1 = A(x+2) + B(x+3)$

$A = -1 \qquad B = 1$

$= \lim_{b\to\infty} -\ln|x+3| + \ln|x+2| \Big|_{-1}^{b}$

$= \lim_{b\to\infty} \ln\left(\frac{x+2}{x+3}\right)\Big|_{-1}^{b}$

$= \lim_{b\to\infty} \ln\left(\frac{b+2}{b+3}\right) - \ln\left(\frac{-1+2}{-1+3}\right)$

1

$= 0 - \ln\frac{1}{2}$

$= -\ln\frac{1}{2}$

$= \ln 2$

(23) $$\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{e^{-x}(e^{2x}+1)}$$

$$= \int \frac{e^x \, dx}{e^{2x}+1}$$

$u = e^x$
$du = e^x dx$

$$= \int \frac{1 \cdot du}{u^2+1} = \tan^{-1} u = \tan^{-1}(e^x)$$

$$\lim_{a \to -\infty} \int_a^0 \frac{du}{u^2+1} + \lim_{b \to \infty} \int_0^b \frac{du}{u^2+1}$$

$$\lim_{a \to -\infty} \tan^{-1}(e^x) \Big|_a^0 + \lim_{b \to \infty} \tan^{-1}(e^x) \Big|_0^b$$

$$\lim_{a \to -\infty} \left(\tan^{-1}(\cancel{e^0}^{\,1}) - \tan^{-1}(\cancel{e^a}^{\,0})\right) + \lim_{b \to \infty} \left(\tan^{-1}(\cancel{e^b}^{\,\infty}) - \tan^{-1}(e^0)\right)$$

$$\tan^{-1}(1) - \tan^{-1}(0) \qquad \tan^{-1}(\infty) - \tan^{-1}(1)$$

$$\frac{\pi}{4} - 0 \quad + \quad \frac{\pi}{2} - \frac{\pi}{4}$$

$$\frac{\pi}{2}$$